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IDENTIFICATION OF HEAT-TRANSFER PROCESSES IN A SUPERSONIC FLOW AROUND A SPHERICALLY BLUNTED CONE USING THE METHODS OF SOLVING THE INVERSE HEAT CONDUCTION PROBLEM

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Solving the heat-protection problems for aircraft involves the determination of the temperature and density of the heat flux on a heated surface. A widely used method of their determination is the solution of the so-called conjugate problems of the mechanics of reacting media [1, 2], which allows one to take into account the heat-and-mass-transfer processes in the gaseous and solid phases and also their mutual effect on each other. As a rule, correct mathematical modeling of conjugate problems requires the use of complex mathematical models with a large set of parameters. Information on a number of model parameters is often either unavailable or not known exactly, and the numerical realization of the models requires extensive computational time resources. Another method of studying the heat effects on aircraft is the method of solving inverse heat-conduction problems (IHCPs) [3–8]. If additional experimental information on the temperature at a certain internal point, line, or body region is known, this method permits ignoring the heat and mass transfer in the gaseous phase. This saves computer time, increases the reliability of results, and sometimes refines the mathematical model of heat and mass transfer in the gaseous phase. However, since IHCPs are often ill-posed, their solution is difficult and requires the development of special regularization algorithms.

Alifanov [9] presents the most detailed analysis of the methods of solution of IHCPs from the viewpoint of their practical use and emphasizes that the iterative regularization method based on gradient algorithms is universal and promising. An algorithm of solving a three-dimensional boundary-value inverse problem for a multilayer hollow spherical segment using the method of iterative regularization is described by Alifanov and Nenarokomov [10]. Great prospects are offered by numerical methods of regularization (especially as applied to the solution of nonlinear multi-dimensional inverse problems based on complex mathematical models). Thus, Kuzin [11] describes a regularized numerical solution of a nonlinear two-dimensional IHCP for a body with rectangular cross section.

The methods of IHCP solution offer an effective tool for studying thermal regimes on the aircraft surface when the only available experimental information is the temperature at some points inside the body or on part of its surface. These methods are a basis for the software of the gauges of unsteady heat fluxes. To obtain a spatial-temporal pattern of the heat-flux density, one usually has to solve a series of one-dimensional IHCPs or use a required number of one-dimensional heat flux gauges along the body contour. In the case where the heat flow along the body contour is substantial (drastically changing heat load along the contour, small radius of the body curvature, highly heat conducting material, etc.), such an approach can lead to large errors in determining the heat-flux density; they can be avoided or reduced using the methods of solving two-dimensional IHCPs.

In this paper, using the methods of solving direct and inverse problems of heat conduction, we determine the heat loads in a supersonic flow around a spherically blunted cone and study the heat flow effect for materials with different thermal diffusivity on the accuracy of determining the temperature and heat-flux density on the aircraft surface. It is shown that, for highly heat conducting materials, neglect of the two-dimensional

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character of heat transfer in the body leads to substantial errors in determining the indicated characteristics. We analyze the influence of the initial temperature uncertainty on the solution of the two-dimensional IHCP.

1. Physical and Mathematical Formulation of Direct and Inverse Problems. Axisymmetric supersonic flow around a spherically blunted hollow cone with shell thickness L is considered. The heat transfer in the body is described in the natural coordinate system by the following heat-conduction equations: for the spherical section,

$$R_N^2 \rho c \frac{\partial T}{\partial t} = \frac{1}{H_1 r}\Big[\frac{\partial}{\partial \xi}\Big(\frac{r}{H_1}\lambda \frac{\partial T}{\partial \xi}\Big) + \frac{\partial}{\partial \bar n}\Big(r H_1 \lambda \frac{\partial T}{\partial \bar n}\Big)\Big], \quad 0 \leqslant \xi \leqslant \xi_1 \tag{1.1}$$

and for the conical section,

$$R_N^2 \rho c \frac{\partial T}{\partial t} = \frac{1}{r}\Big[\frac{\partial}{\partial \xi}\Big(r\lambda \frac{\partial T}{\partial \xi}\Big) + \frac{\partial}{\partial \bar n}\Big(r \lambda \frac{\partial T}{\partial \bar n}\Big)\Big], \quad \xi_1 < \xi \leqslant \xi_l \tag{1.2}$$

with the initial and boundary conditions

$$t = 0: \qquad T = T_{\rm in}(\xi, \bar n); \tag{1.3}$$

$$\bar n = 0: \qquad \frac{\alpha}{c_p}(H_r - h_w)\Big|_{\bar n = -0} - \varepsilon\sigma T_w^4\Big|_{\bar n=-0} = -\frac{\lambda}{R_N}\frac{\partial T}{\partial \bar n}\Big|_{\bar n = +0}; \tag{1.4}$$

$$\bar n = \frac{L}{R_N}: \quad \frac{\partial T}{\partial \bar n} = 0; \tag{1.5}$$

$$\xi = 0: \quad \frac{\partial T}{\partial \xi} = 0; \tag{1.6}$$

$$\xi = \xi_1: \quad \frac{\lambda}{H_1}\frac{\partial T}{\partial \xi}\Big|_{\xi=\xi_1-0} = \lambda\frac{\partial T}{\partial \xi}\Big|_{\xi=\xi_1+0}; \tag{1.7}$$

$$\xi = \xi_l: \quad \frac{\partial T}{\partial \xi} = 0. \tag{1.8}$$

Here and below T is the temperature, $\bar n = n/R_N$ and $\xi = s/R_N$ are the crossflow and streamwise coordinates, t is time, $r = r_w/R_N - \bar n\cos(\pi/2 - \xi)$ and $H_1 = 1 - \bar n$ are the Lamé coefficients, R_N is the spherical bluntness radius, $\xi_1 = \pi/2 - \beta$, β is the cone angle, $r_w/R_N = \sin\xi$ for the spherical section, $r_w/R_N = \sin\xi_1 + (\xi - \xi_1)\sin\beta$ for the conical section, ρ is the density, c_p is the specific heat of the gas, c is the heat capacity of the solid body, λ is the heat conductivity, α is the heat-transfer coefficient, H_r is the recovery enthalpy, h_w is the gas enthalpy on the wall, ε is the emissivity factor, σ is the Stefan–Boltzmann constant, and $q_w = (\alpha/c_p)(H_r - h_w)$ and $Q_w = q_w - \varepsilon\sigma T_w^4$ are the convective and overall heat fluxes from the gaseous phase.

We consider a mixed boundary layer flow: laminar in the spherical section around the stagnation point and turbulent on the spherical bluntness periphery and on the cone.

The value of H_r for the laminar flow regime is determined from the formula

$$H_r = H_{e0}[(P_e/P_{e0})^{(\gamma-1)/\gamma} + \mathrm{Pr}^{1/2}(U_e/v_m)^2] \tag{1.9}$$

$$\Big(H_{e0} = h_\infty[1 + ((\gamma-1)/2)\mathrm{M}_\infty^2], \quad U_e/v_m = [1 - (P_e/P_{e0})^{(\gamma-1)/\gamma}]^{1/2}, \quad v_m = \sqrt{2H_{e0}}\Big),$$

and, for the turbulent flow regime, it is determined from the equation

$$H_r = H_{e0}[(P_e/P_{e0})^{(\gamma-1)/\gamma} + \mathrm{Pr}^{1/3}(U_e/v_m)^2]. \tag{1.10}$$

The heat-transfer coefficients are found using the data of Zemlyanskii and Stepanov [12]. On the sphere periphery for the laminar flow regime, we have

$$(\alpha/c_p)(\xi) = (0.55 + 0.45\cos 2\xi)(\alpha/c_p)(0),$$
$$(\alpha/c_p)(0) \approx 1.05 U_\infty^{1.08}(\rho_\infty/R_N)^{1/2} \qquad (U_\infty\,[\mathrm{km/sec}],\ \rho_\infty\,[\mathrm{kg\cdot sec^2/m^4}],\ \text{and}\ \ R_N\,[\mathrm{m}]), \tag{1.11}$$

and for the turbulent regime, we have

$$(\alpha/c_p)(\xi) = (3.75\sin\xi - 3.5\sin^2\xi)(\alpha/c_p)(\xi_*),$$
$$(\alpha/c_p)(\xi_*) \approx 16.4U_\infty^{1.25}\rho_\infty^{0.8}/[R_N^{0.2}(1 + h_w/H_{e0})^{2/3}]. \tag{1.12}$$

In the conical section for the turbulent flow regime,

$$(\alpha/c_p)(\xi) = \{2.2(P_e/P_{e0})(U_e/v_m)/[k^{0.4}(r_w/R_N)^{0.2}]\}(\alpha/c_p)(\xi_*), \quad k = (\gamma - 1 + 2/\mathrm{M}_\infty^2)/(\gamma + 1). \tag{1.13}$$

The function $P_e/P_{e0}(\xi)$ for the sphere is calculated using the formula [13]

$$P_e/P_{e0}(\xi) = 1 - 1.17\sin^2\xi + 0.225\sin^6\xi; \tag{1.14}$$

the tabulated data for the cone are borrowed from [14]. The enthalpy for air is $h_w = 965.5\,T_w + 0.0735T_w^2$. Here and below, U_e is the velocity, P is the pressure, γ is the ratio of specific heats, M is the Mach number, and Pr is the Prandtl number; the subscripts "in" and "fin" denote the initial and final states, w the surface $\bar{n} = 0$, e and $e0$ the conditions at the outer edge of the boundary layer and at the stagnation point, respectively, and ∞ the free-stream conditions.

The direct heat-conduction problem (DHCP) consists in finding the function $T(\xi, \bar{n}, t)$ that satisfies Eqs. (1.1) and (1.2) in the open region $D = \{(\xi, \bar{n}, t)\colon 0 < \xi < \xi_l,\ 0 < \bar{n} < L/R_N, \text{ and } 0 < t \leqslant t_{\text{fin}}\}$ and initial and boundary conditions (1.3)–(1.8) with relations (1.9)–(1.14) and is continuous together with its derivatives $\partial T(\xi, \bar{n}, t)/\partial\xi$ and $\partial T(\xi, \bar{n}, t)/\partial\bar{n}$ in the closed region $\bar{D}$.

When the heat flux from the gaseous phase is not known and we have to determine the temperature field $T(\xi, \bar{n}, t)$ in the region $\bar{D}$ and the density of the overall $Q_w(\xi, t)$ and convective $q_w(\xi, t)$ heat fluxes over the surface $\bar{n} = 0$ from the known temperature at the line $\bar{n} = L/R_N$

$$T(\xi, L/R_N, t) = T_{\text{bound}}(\xi, t), \tag{1.15}$$

we obtain a two-dimensional inverse heat-conduction problem.

2. Algorithms of Solving the Direct and Inverse Problems. Let us write the heat-conduction equations (1.1) and (1.2) in the general form

$$F_3(T(\xi, \bar{n}, t), \xi, \bar{n})\,\frac{\partial T}{\partial t} = \frac{\partial}{\partial\bar{n}}\left[F_1(T(\xi, \bar{n}, t), \xi, \bar{n})\,\frac{\partial T}{\partial\bar{n}}\right] + \frac{\partial}{\partial\xi}\left[F_2(T(\xi, \bar{n}, t), \xi, \bar{n})\,\frac{\partial T}{\partial\xi}\right],$$

where F_i are determined from the above formulation of the problem.

The splitting method is used for solving the two-dimensional DHCP [15]. The one-dimensional heat-conduction equations obtained by splitting in each time half-step are solved by the iteration-interpolation method (IIM) [16] with iterations over the coefficients. In the first step, the calculations are performed in the $\bar{n}$ direction, and in the second step in the ξ direction. A special difference equation obtained on the basis of IIM [17] is used for temperature calculations in the ξ direction at the sphere–cone junction.

The solution of the two-dimensional IHCP is based on the algorithm from [11]. Unlike in [11], a variable step along ξ is used in the general case, and Tikhonov's regularization functional is supplemented by the term $\delta(k_3||\partial T/\partial\xi||^2 + k_4||\partial^2T/\partial\xi^2||^2)$, which ensures regularization of the solution with respect to the variable ξ. Here $||\cdot||$ is the norm in the space of functions integrated with the square $L_2[0, t_{\text{fin}}]$, δ is the regularization parameter, and $k_3 > 0$ and $k_4 > 0$ are non-negative numbers. For an unknown error of the initial temperature $T_{\text{bound}}(\xi, t)$, the optimal approximation is chosen using the principle of quasi-optimal parameters, and for a known error, it is chosen using the principle of residuals.

3. Numerical Results. The influence of heat flow along the contour of a spherically blunted cone on the accuracy of determining the temperature and heat-flux density on the heated surface $\bar{n} = 0$ was numerically studied using the methods of solving direct and inverse problems of heat conduction. Using the above algorithms, we wrote codes for DHCP and IHCP calculations in FORTRAN for an IBM PC AT-386. We study materials with a wide range of thermophysical characteristics λ, ρ, and c: a highly heat-conducting material [copper, $\lambda = 386$ W/(m · K), $\rho = 8950$ kg/m^3, and $c = 376$ J/(kg · K)], a low heat-conducting material [carbon plastic, $\lambda = 0.75$ W/(m · K), $\rho = 1350$ kg/m^3, and $c = 1062$ J/(kg · K)], and a material with

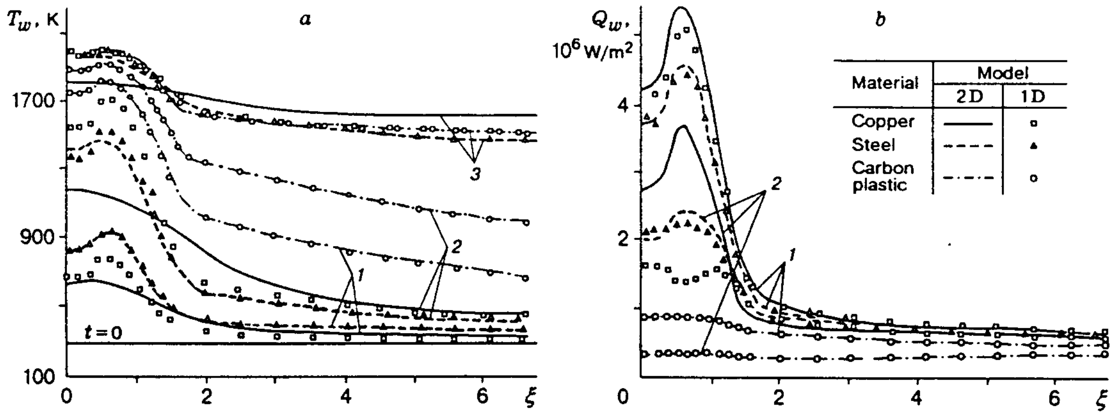

Fig. 1

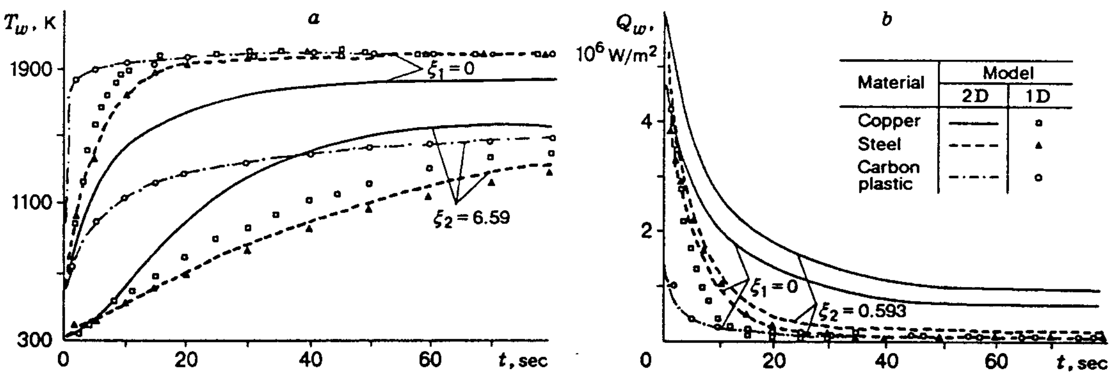

Fig. 2

intermediate thermophysical characteristics [steel, $\lambda = 20$ W/(m · K), $\rho = 7800$ kg/m^3, $c = 600$ J/(kg · K)]. The following parameters were used in the calculations: $R_N = 0.0185$ m, $L = 0.005$ m, $\beta = 5°$, $\xi_l = 6.59$, $M_\infty = 6$, $U_\infty = 2078$ m/sec, $\rho_\infty = 0.02$ kg · sec^2/m^4, $\gamma = 1.4$, Pr $= 0.72$, $\varepsilon = 0.7$, $H_{e0} = 2.46 \cdot 10^6$ m^2/sec^2, and $T_{\text{in}} = 300$ K.

The results of the solution of the DHCP are presented in Figs. 1 and 2. Figure 1a and b shows the distributions of temperature T_w and heat-flux density Q_w along the body contour for times $t = 1$ and 5 sec (curves 1 and 2), and for time $t = t_{\text{st}}$ (curves 3) (t_{st} is the time required for the attainment of the steady regime) for copper, steel, and carbon plastic, which were obtained within the framework of two-dimensional (curves) and one-dimensional (points) mathematical models. As is evident from Fig. 1, the use of the heat-conducting material leads to a decrease in surface temperature by several hundred degrees in the initial period of time. When the steady regime is attained, the decrease in the maximum surface temperature on the spherical section is about 200 K for copper. The surface temperature distribution along the contour, $T_w(\xi)$, is monotonic at various times, and the curves level off as $t \to \infty$ (solid curve 3). This process is related to heat flow to the conical part of the body and its subsequent re-emission from the body surface.

The distributions $T_w(\xi)$ for steel and carbon plastic are in qualitative agreement with the behavior of the convective heat flux density along the body contour, $q_w(\xi)$. As might be expected, two-dimensionality is insignificant for carbon plastic, weakly manifested for steel, and important for copper. In the latter case, for one-dimensional and two-dimensional formulations of the problem, the results are qualitatively and quantitatively different. Thus, for copper at $t = 5$ sec, the maximum difference between temperatures calculated by the one-dimensional and two-dimensional models is ~550 K (the relative calculation error is $\varepsilon_{\text{calc}} \sim 50\%$), and the heat flux-density difference is ~$2.2 \cdot 10^6$ W/m^2 ($\varepsilon_{\text{calc}} \sim 60\%$). Note that, for the

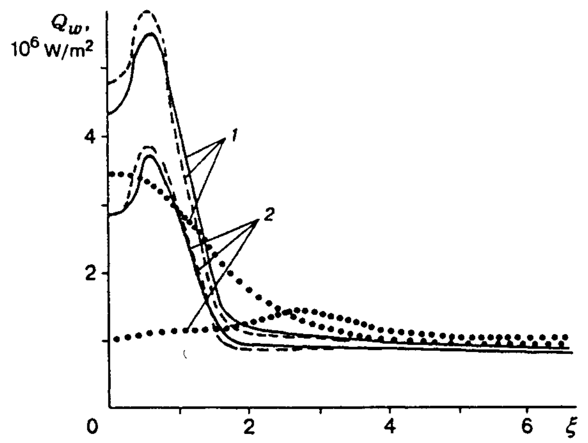

Fig. 3

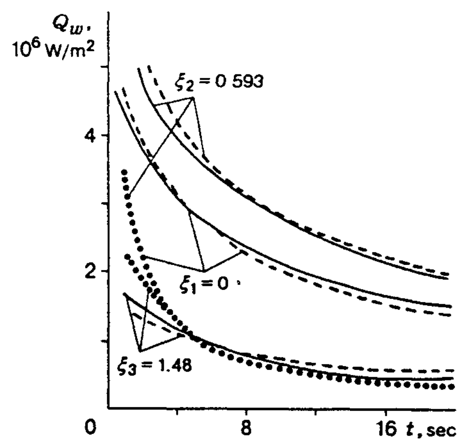

Fig. 4

one-dimensional formulation of the problem using the boundary conditions for the steady-state regime, the solution is independent of the heat conductivity of the material and coincides with the radiation temperature of the surface found from the condition $q_w = \varepsilon\sigma T_w^4$.

The temperature T_w and the heat-flux density Q_w versus time t are presented in Fig. 2a and b (notation coincides with Fig. 1). Figure 2a shows the function $T_w(\xi_i, t)$ for surface points $\bar{n} = 0$ with the coordinates $\xi_1 = 0$ and $\xi_2 = 6.59$, and Fig. 2b shows the function $Q_w(\xi_i, t)$ with the coordinates $\xi_1 = 0$ and $\xi_2 = 0.593$.

The results of solving the one-dimensional IHCPs in Fig. 2b are presented for $\xi = 0$. The surface temperature increases in time due to material heating and reaches an asymptotic value, whereas the heat-flux density decreases. At the same times, the highest temperature and the lowest heat-flux density for $\xi = 0$ are observed for the material with the lowest heat conductivity — carbon plastic. The fastest attainment of the asymptotic temperature value is also typical of this material. Neglect of heat flow along the body contour for copper leads to a more rapid growth in T_w and attainment of the asymptotic value at the frontal stagnation point and to a slower temperature growth on the periphery ($\xi = \xi_l$). Two-dimensionality effects are insignificant for steel and especially for carbon plastic. The temperature T_w for $\xi = 0$ for copper determined with or without allowance for heat flow along the body contour can differ by more than 400 K for $t = 10$ sec, and the heat-flux density Q_w can differ by several times.

Thus, the analysis of Figs. 1 and 2 shows that, when solving the DHCP, one should take into account the two-dimensionality of heat-transfer processes in samples made of highly heat-conducting materials. In this case, one should expect a considerable influence of heat flow in determining the heat-flux density and surface temperature by the methods of solving inverse problems. In this connection, a copper sample will be used as the object of further investigation.

Figures 3–6 show the results of the solution of the IHCP. The initial "experimental" information for solving both the one-dimensional and two-dimensional IHCPs was the temperature at the back surface of the shell $T_{\text{bound}}(\xi, t)$ obtained by solving the two-dimensional DHCP (1.1)–(1.8) with relations (1.9)–(1.14). The functions $T_w(\xi, t)$ and $Q_w(\xi, t)$ obtained by solving this DHCP will be further considered an exact solution of the IHCP.

Figure 3 shows the heat-flux-density distribution along the body contour for times $t = 1$ and 5 sec (curves 1 and 2). The exact solution of the two-dimensional IHCP is shown in Figs. 3 and 4 by solid curves, the dashed curves describe the numerical solution of the two-dimensional IHCP, and the dotted curves show the solution of a series of one-dimensional problems along the body contour. For numerical experiments, the number of nodes of the difference grid for the variables ξ, $\bar{n}$, and t was 11, 11, and 100, respectively. Figure 3 shows that the numerical solution of the two-dimensional IHCP is stable and agrees well with the exact solution. At the same time, the value of Q_w obtained by solution of the series of one-dimensional IHCPs can

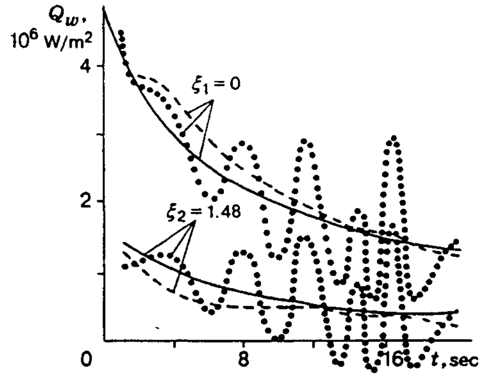

Fig. 5

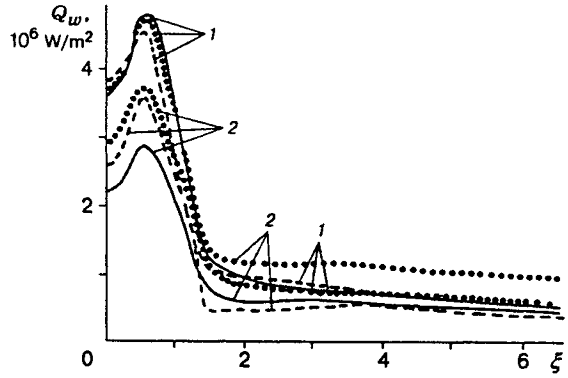

Fig. 6

differ from the exact value by more than a factor of 1.5. Even the qualitative behavior of $Q_w(\xi)$ on the sphere changes (becomes monotonic).

Figure 4 shows the curves of $Q_w(\xi_i, t)$ for points of the heated surface with the coordinates $\xi_1 = 0$, $\xi_2 = 0.593$, and $\xi_3 = 1.48$. We see that the curves of $Q_w(\xi_i, t)$ found from the solution of the two-dimensional IHCP are in good agreement with the exact results, whereas those obtained by solution of the one-dimensional IHCP can differ from the exact solutions by several times.

It follows from Figs. 3 and 4 that, first, the algorithm used for the solution of the two-dimensional IHCP allows us to reconstruct $Q_w(\xi, t)$ with fairly good accuracy. Second, the use of one-dimensional IHCPs with $\xi \leqslant 4$ leads to large errors in determining $Q_w(\xi, t)$, and, whence, it is necessary to use two-dimensional IHCPs. At the same time, the solution of the one-dimensional IHCP yields acceptable accuracy for peripheral sections of the conical surface ($\xi > 4$), as is evident from the weak variation in the heat-flux density along the body contour in this region. This result could be expected on the basis of the results of the solution of the direct problem, which are presented in Fig. 1b.

Figures 5 and 6 show the effect of the initial temperature errors on the solution of the two-dimensional IHCP. Curves 1 and 2 in Fig. 6 correspond to $t_1 = 2$ sec and $t_2 = 8$ sec, respectively. Disturbances distributed uniformly in time with a 3% maximum deviation from the current temperature value were superimposed on the temperature $T_{\text{bound}}(\xi, t)$. Here the solid curves show the exact solution of the IHCP, the dotted curves describe the numerical solution of the IHCP ($\delta = 0$) without smoothing the initial temperature, and the dashed curves show the numerical solution of the IHCP ($\delta = 0$) with preliminary smoothing of the initial temperature using Tikhonov's regularization method with the regularization parameter chosen on the basis of the residual principle [18]. We see that the dependence $Q_w(t)$ found from the solution of the IHCP without temperature smoothing has an explicit unstable character and can take even negative values. The solution of the IHCP obtained with prior smoothing of the initial temperature is stable and agrees well with the exact solution.

Thus, by solving direct problems, it is shown that the use of highly heat-conducting materials is promising for decreasing the maximum surface temperatures. The methods of solving the inverse problems of determining the heat-flux density are implemented. The influence of heat flow along the streamwise coordinate on the desired quantity $Q_w(\xi, t)$ is analyzed.

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